EE-208-F:
Fundamentals of Electromagnetics

LECTURE 3

Topics

- Coordinate systems
- Cartesian coordinates,
- Circular coordinates,
- Cylindrical coordinates,

VECTOR REPRESENTATION

3 PRIMARY COORDINATE SYSTEMS:

- RECTANGULAR
- CYLINDRICAL
- SPHERICAL

Choice is based on symmetry of problem

<u>Examples:</u>

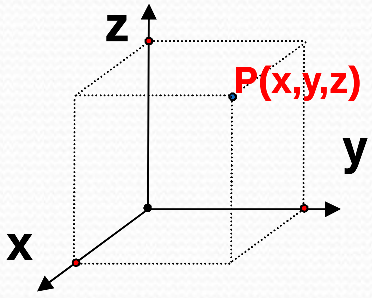
Sheets - RECTANGULAR

Wires/Cables - CYLINDRICAL

Spheres - SPHERICAL

Orthogonal Coordinate Systems: (coordinates mutually perpendicular)

Cartesian Coordinates

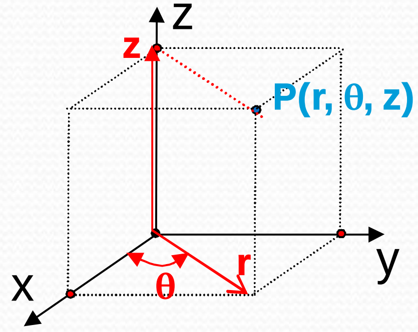


Rectangular Coordinates

P (x,y,z)

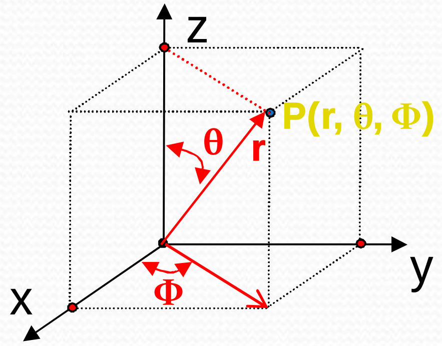
Cylindrical Coordinates

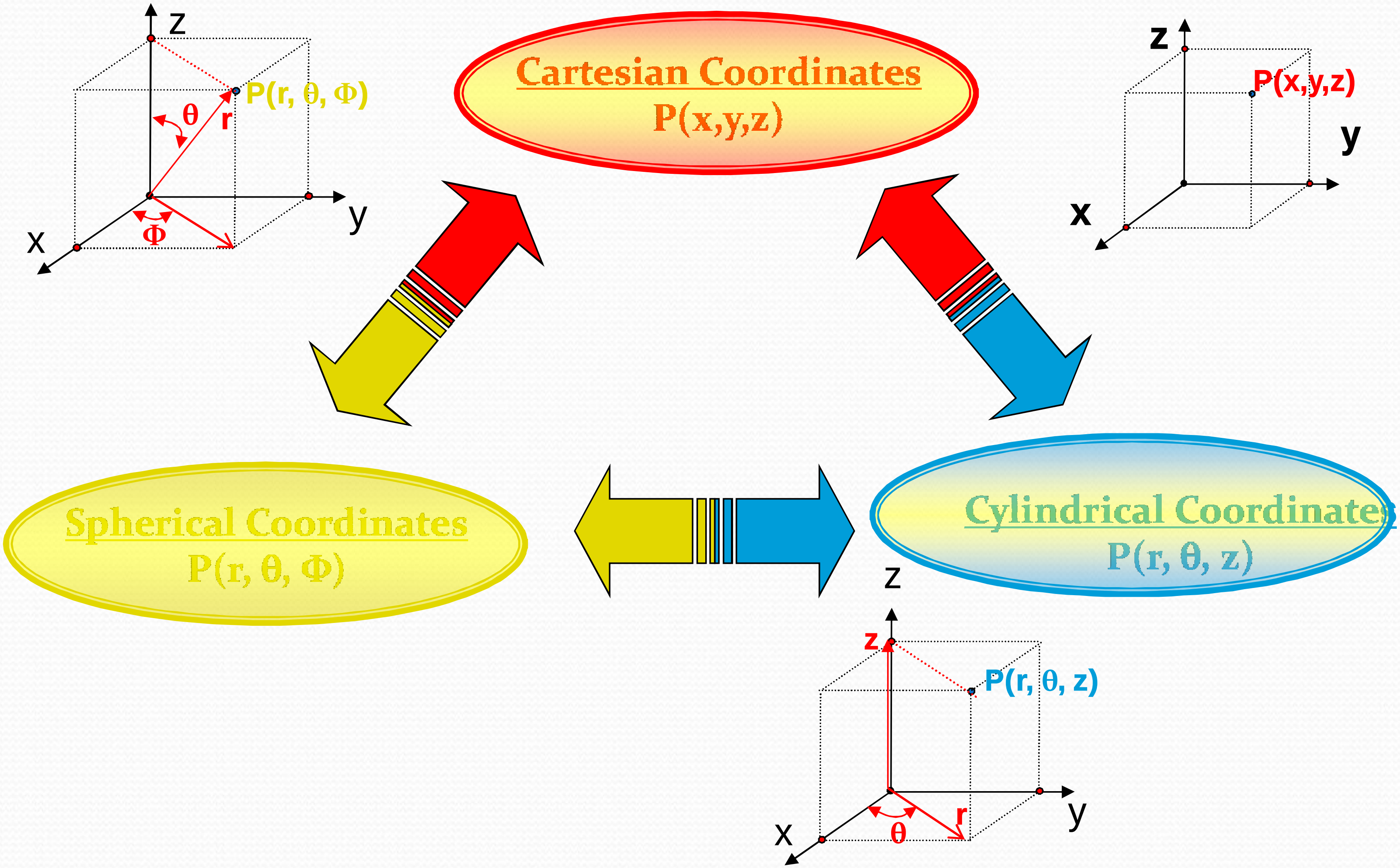
P (r, Θ, z)



Spherical Coordinates

P (r, Θ, Φ)



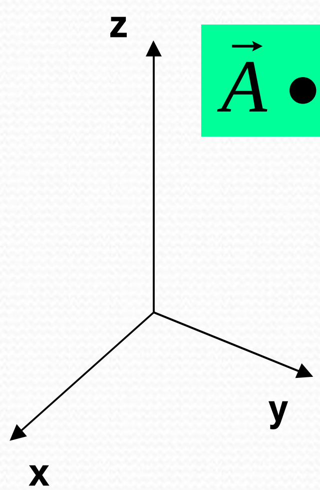

Cartesian Coordinates
P(x,y,z)
Spherical Coordinates
P(r, θ, Φ)
Cylindrical Coordinates
P(r, θ, z)
z
y
x
P(r, θ, Φ)
θ
r
Φ
z
y
x
P(x,y,z)
z
y
x
z
P(r, θ, z)
θ
r

VECTOR NOTATION

VECTOR NOTATION:

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

Rectangular or Cartesian Coordinate System



$$\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Dot Product

(SCALAR)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cross Product

(VECTOR)

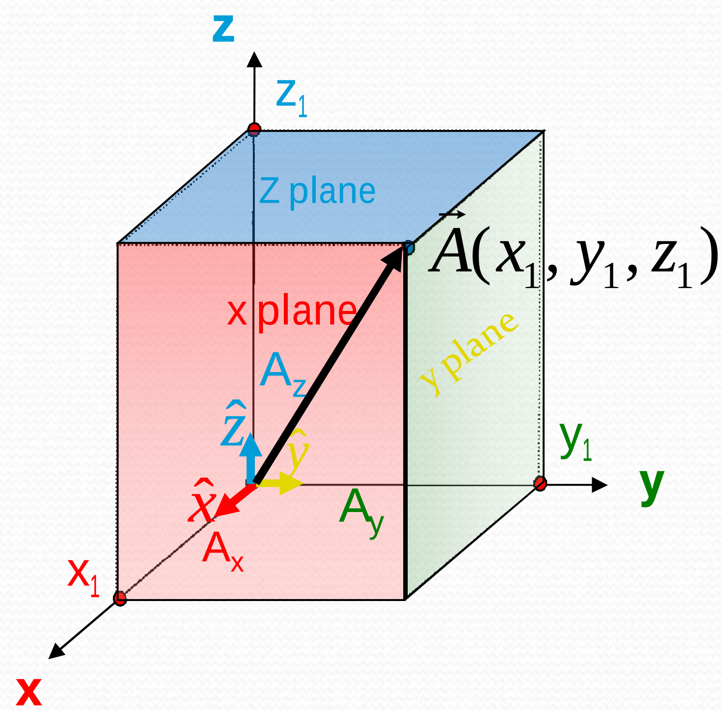
$$\left|\vec{A}\right| = \left(A_x^2 + A_y^2 + A_z^2\right)^{\frac{1}{2}}$$

Magnitude of vector

Cartesian Coordinates
(x, y, z)

Vector representation

$$\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$



Magnitude of A

$$|\vec{A}| = \sqrt[+]{\vec{A}\cdot\vec{A}} = \sqrt[+]{A_x^2 + A_y^2 + A_z^2}$$

Position vector A

$$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$$

Base vector properties

$$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$$

$$\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$$

$$\hat{x}\times\hat{y} = \hat{z}$$

$$\hat{y}\times\hat{z} = \hat{x}$$

$$\hat{z}\times\hat{x} = \hat{y}$$

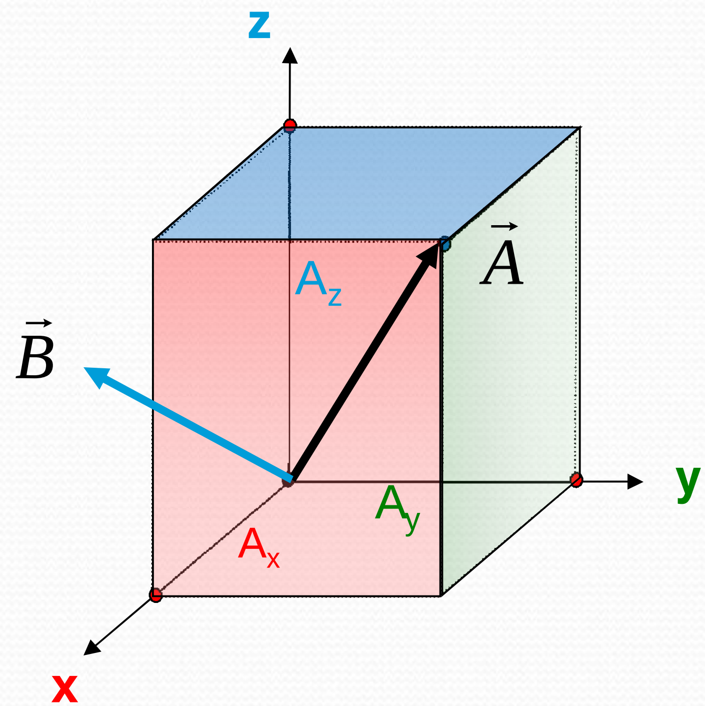
Cartesian Coordinates

Dot product:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross product:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



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VECTOR REPRESENTATION: CYLINDRICAL COORDINATES

UNIT VECTORS:

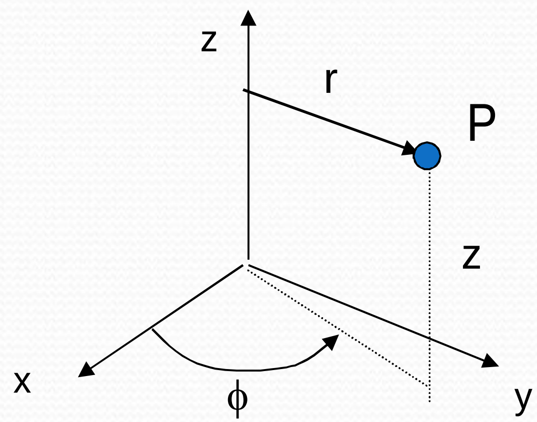
$$\left(\hat{a}_r \quad \hat{a}_\phi \quad \hat{a}_z\right)$$

Cylindrical representation uses: r, ϕ, z

$$\vec{A} = A_r\hat{a}_r + A_\phi\hat{a}_\phi + A_z\hat{a}_z$$

$$\vec{A} \bullet \vec{B} = A_rB_r + A_\phi B_\phi + A_zB_z$$

Dot Product
(SCALAR)



VECTOR REPRESENTATION: SPHERICAL COORDINATES

UNIT VECTORS:

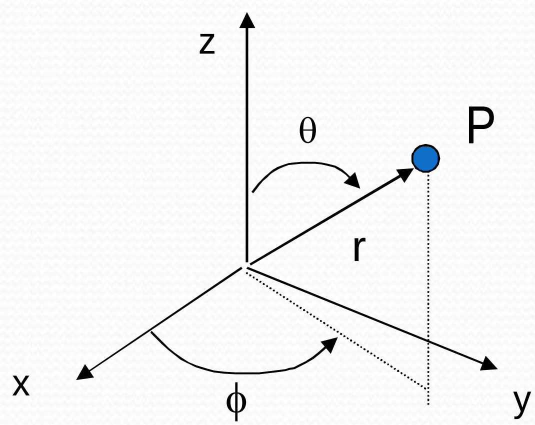
$$\left(\hat{a}_r \quad \hat{a}_\theta \quad \hat{a}_\phi\right)$$

Spherical representation uses: r, θ, ϕ

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

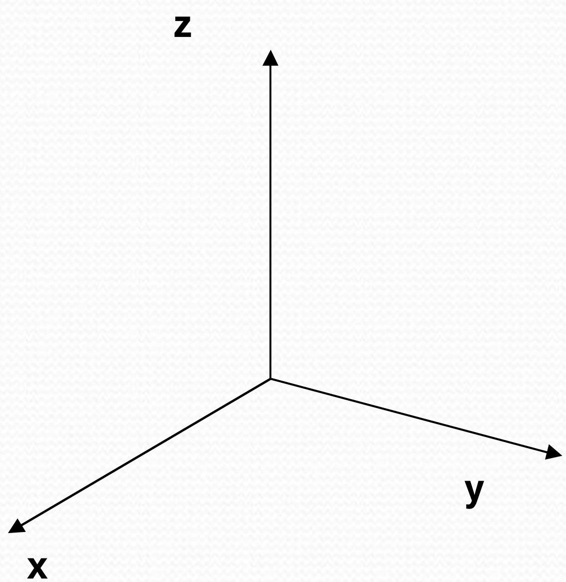
$$\vec{A} \bullet \vec{B} = A_r B_r + A_\theta B_\theta + A_\phi B_\phi$$

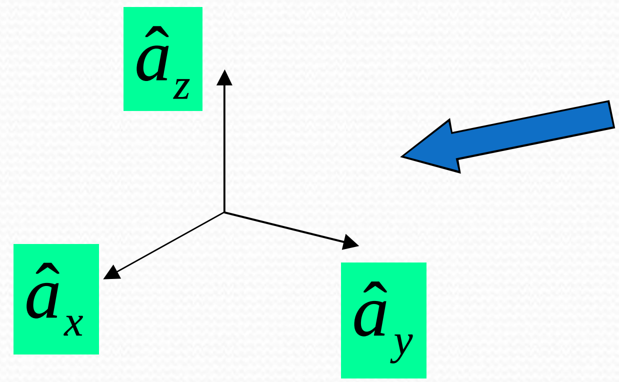
Dot Product
(SCALAR)



VECTOR REPRESENTATION: UNIT VECTORS

Rectangular Coordinate System





Unit Vector Representation for Rectangular Coordinate System

The Unit Vectors imply :

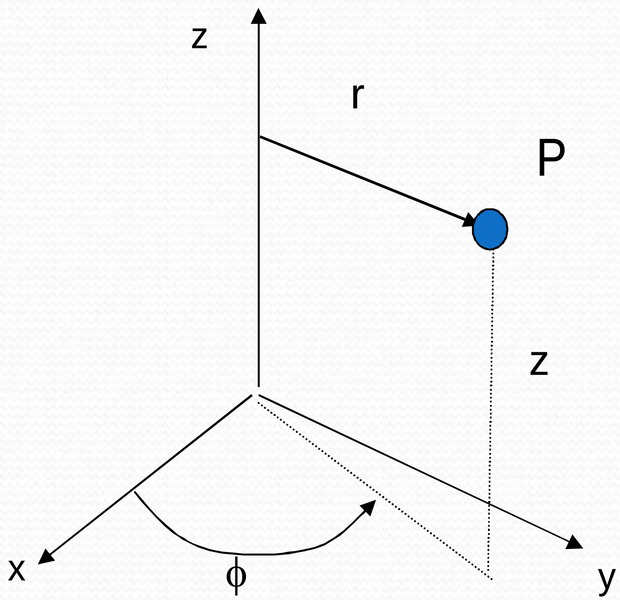
$\hat{a}_x$ → Points in the direction of increasing **x**

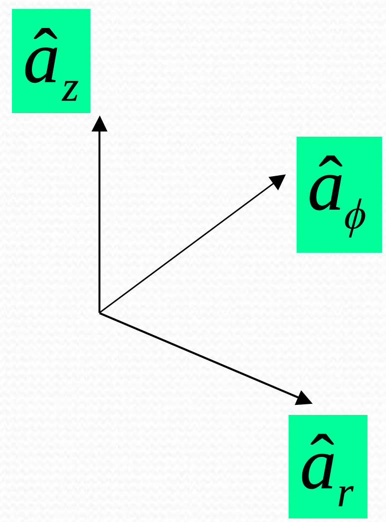
$\hat{a}_y$ → Points in the direction of increasing **y**

$\hat{a}_z$ → Points in the direction of increasing **z**

VECTOR REPRESENTATION: UNIT VECTORS

Cylindrical Coordinate System





The Unit Vectors imply :

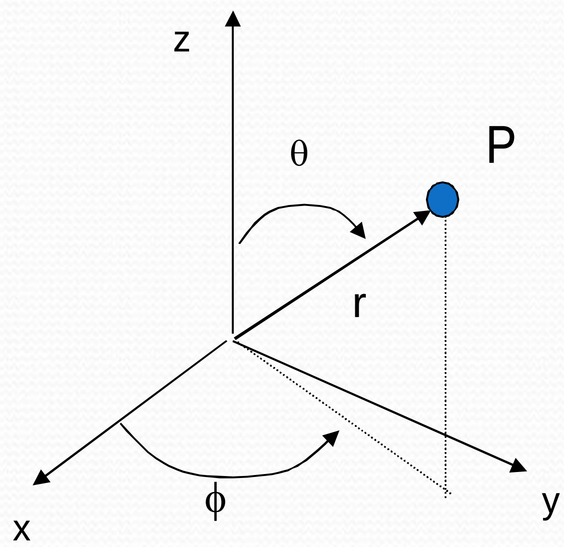
$\hat{a}_r$ ➡ Points in the direction of increasing **r**

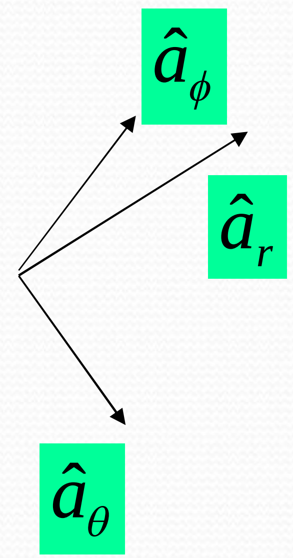
$\hat{a}_\phi$ ➡ Points in the direction of increasing φ

$\hat{a}_z$ ➡ Points in the direction of increasing **z**

VECTOR REPRESENTATION: UNIT VECTORS

Spherical Coordinate System





The Unit Vectors imply :

$\hat{a}_r$ ➡ Points in the direction of increasing **r**

$\hat{a}_\theta$ ➡ Points in the direction of increasing θ

$\hat{a}_\phi$ ➡ Points in the direction of increasing φ

VECTOR REPRESENTATION: UNIT VECTORS

Summary

RECTANGULAR Coordinate Systems	CYLINDRICAL Coordinate Systems	SPHERICAL Coordinate Systems
$\left(\hat{a}_x \quad \hat{a}_y \quad \hat{a}_z\right)$	$\left(\hat{a}_r \quad \hat{a}_\phi \quad \hat{a}_z\right)$	$\left(\hat{a}_r \quad \hat{a}_\theta \quad \hat{a}_\phi\right)$

NOTE THE ORDER!

r, ϕ, z $\qquad$ r, θ, ϕ

Note: We do not emphasize transformations between coordinate systems

METRIC COEFFICIENTS

Unit is in "meters"

1. Rectangular Coordinates:

When you move a small amount in **x**-direction, the distance is **dx**

In a similar fashion, you generate **dy** and **dz**